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*Letter*

# 1/2+ uudds̄ pentaquark and instanton-induced forces

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**Abstract.** The mass of the $1/2^+$ $uudd\bar{s}$ pentaquark is calculated within the framework of a semirelativistic effective Hamiltonian approach to QCD with instanton-induced forces, using a diquark picture. This approximation allows a correct treatment of the confinement, assumed here to be a Y-junction. With the $[ud]$ diquark mass fitted on the $\Lambda$-baryon, the ground-state pentaquark is found around 2.2 GeV.

**PACS.** 12.39.Pn Potential models – 12.39.Ki Relativistic quark model – 14.20.-c Baryons (including antiparticles)

Recent experiments have reported the existence of a very narrow peak in $K^+n$ and $K^0p$ invariant mass distributions at 1.540 GeV [1], which is interpreted as a $uudd\bar{s}$ pentaquark [2]. Quantum numbers are not known yet but a $J^P = 1/2^+$ assignment is preferred. Models using the diquark approximation have been proposed to explain the properties of this state. In ref. [3], a good value is obtained for the mass, but the model does not take into account the full confinement dynamics. In refs. [4,5], the confinement is correctly taken into account, but pentaquark masses are found above 2 GeV. In this work, we will study the influence on the pentaquark mass of a possible residual interaction stemming from instanton-induced interaction, which is the only spin-dependent interaction capable of giving supplementary attraction in the diquark picture considered in refs. [4,5].

The dominant interaction in a pentaquark is certainly the confinement. As this multiquark is a complicated five-body system, we will assume that it can be reduced to a three-body system, for which a realistic confinement potential can be built. We will assume, as in the work of Jaffe and Wilczek [3], that quarks can form diquark clusters inside the pentaquark.

All short-range interactions available between quarks, one-gluon exchange [6], Goldstone-boson exchange [7] and instanton-induced [8] interactions, predict that the most probable diquark which can be formed is the $[ud]$ pair in colour $\bar{3}$ representation with vanishing spin and isospin. In this case, the pentaquark considered here can be viewed as an antibaryon-like system $DD\bar{s}$, where $D = [ud]$, and the confinement can be simulated by a Y-junction

$$V_{\rm Y} = a \min_{\mathbf{r}_0} \sum_{i=1}^{3} |\mathbf{r}_i - \mathbf{r}_0|, \qquad (1)$$

where $a$ is the string tension. In this work we will use a very good approximation of this potential, free from three-body complications, given by [9]

$$\widetilde{V}_{\rm Y} = \frac{1}{2}\left(V_\Delta + V_{\rm CM}\right), \qquad (2)$$

where

$$V_\Delta = \frac{1}{2}\, a \sum_{i<j=1}^{3} |\mathbf{r}_i - \mathbf{r}_j| \quad \text{and} \quad V_{\rm CM} = a \sum_{i=1}^{3} |\mathbf{r}_i - \mathbf{r}_{\rm CM}|, \qquad (3)$$

in which $\mathbf{r}_{\rm CM}$ is the centre-of-mass coordinate.

We use an effective QCD Hamiltonian derived in ref. [10], but with all its auxiliary fields eliminated, as defined in ref. [11]

$$H_0 = \sum_{i=1}^{3} \sqrt{\mathbf{p}_i^2 + m_i^2} + \widetilde{V}_{\rm Y} - \frac{2}{3} \sum_{i<j=1}^{3} \frac{\alpha_{\rm S}}{|\mathbf{r}_i - \mathbf{r}_j|}, \qquad (4)$$

where $\alpha_{\rm S}$ is the strong coupling constant. The particle self-energy is also taken into account and appears as a

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contribution depending on the constituent particle mass

$$M = M_0 + \sum_{i=1}^{3} \frac{C(\mathbf{s}_i, m_i, a, \delta)}{\langle \sqrt{\mathbf{p}_i^2 + m_i^2}\, \rangle}, \tag{5}$$

where $C(\mathbf{s}_i, m_i, a, \delta)$ is a negative contribution for a fermion and vanishes for a boson [10]. The inverse gluonic correlation length $\delta$ is around 1 GeV.

The mass of the diquark $D$ will be fitted on baryon spectra, considered here as $Dq$ systems. The equivalent two-body energy operator is given by

$$H_0 = \sum_{i=1}^{2} \sqrt{\mathbf{p}_i^2 + m_i^2} + a\, r - \frac{4}{3} \frac{\alpha_{\mathrm{S}}}{|\mathbf{r}_1 - \mathbf{r}_2|}, \tag{6}$$

$$M = M_0 + \sum_{i=1}^{2} \frac{C(\mathbf{s}_i, m_i, a, \delta)}{\langle \sqrt{\mathbf{p}_i^2 + m_i^2}\, \rangle}. \tag{7}$$

To consider the nucleon as a pure $Dn$ state ($n$ stands for $u$ or $d$) or the $\Lambda$-baryon as a pure $Ds$ state is probably not a very good approximation [12]. But our aim is just to obtain a reasonable estimation for the mass of the $[ud]$ diquark.

One-gluon exchange and Goldstone-boson exchange potentials predict no interaction between the two diquarks, and each diquark and the $\bar{s}$-quark, since the two diquarks are spin singlets. This is not the case for the instanton-induced interaction. At the nonrelativistic limit, the instanton interaction between two quarks is given by [13]

$$V_{qq'} = -2 \left( g\, P^{[nn]} + g'\, P^{[ns]} \right) \left( P^{S=1} P_6^C + 2\, P^{S=0} P_{\bar{3}}^C \right) \delta(\mathbf{r}\,), \tag{8}$$

where $P^{[qq']}$ are projectors on flavour antisymmetrical $qq'$ states, $P^{S=x}$ are projectors on spin $x$ states, and $P_y^C$ are projectors on colour representation of dimension $y$. $g$ and $g'$ are two-dimensioned constants. The instanton interaction between a quark and an antiquark is given by

$$V_{q\bar{q}} = \hat{g} \left( \frac{3}{2} P^{S=1} P_8^C + P^{S=0} \left( \frac{1}{2} P_8^C + 8\, P_1^C \right) \right) \delta(\mathbf{r}\,), \tag{9}$$

where $\hat{g}$ is a flavour projector operator. Its value is $-g$ for $u\bar{d}$ and $d\bar{u}$, and $-g'$ for $n\bar{s}$ and $s\bar{n}$. This operator also couples isospin 0 $q\bar{q}$ states. The spatial dependence of these potentials is singular. We have chosen to regularize it by replacing the $\delta$ distribution by the Gaussian [13]

$$\delta(\mathbf{r}\,) \to \rho(r) = \frac{1}{(\gamma\sqrt{\pi})^3}\, e^{-r^2/\gamma^2}\, \delta_{L,0}, \tag{10}$$

where $\gamma$ can be interpreted as the range of this interaction. Let us remark that this interaction can be stronger for a $q\bar{q}$ pair than for a $qq$ pair. Consequently, the diquark approximation is questionable in a pentaquark; but it is the only way to take into account the confinement with a realistic potential.

With the Hamiltonian (6)-(7)-(9) and the following parameters: $m_n = 0.200$ GeV, $m_s = 0.320$ GeV, $a = 0.15$ GeV$^2$, $\alpha_{\mathrm{S}} = 0.39$, and $\delta = 1$ GeV, good masses are found for the light $S = 1$ mesons. The results are not sensitive to the parameter $\gamma$, so we will present results only for the following instanton parameters: $\gamma = 2$ GeV$^{-1}$ [13], $g = 10.6$ GeV$^{-2}$, $g' = 7.4$ GeV$^{-2}$. In this case, good masses are found for the pseudoscalar mesons.

If we take into account this instanton-induced interaction for the nucleon, considered here as a $Dn$ system, we find with the Hamiltonian (6)-(7)-(8)

$$\langle N|V_{\mathrm{inst}}|N\rangle = -\frac{1}{2}\, g\, \rho(r_{Dn})\, \delta_{L,0}. \tag{11}$$

For the $\Lambda$-baryon considered here as a $Ds$ system, we find

$$\langle \Lambda|V_{\mathrm{inst}}|\Lambda\rangle = -g'\, \rho(r_{Ds})\, \delta_{L,0}. \tag{12}$$

Contributions of the instanton-induced interaction for light baryons in a $SU(3)$ flavour scheme can be found in ref. [14]. These results are obtained by summing the two instanton interactions due to $uq$ and $dq$ pairs, and taking $r_{uq} = r_{uq} = r_{Dq}$. With a diquark mass $m_D = 0.330$ GeV, we find $m_\Lambda = 1.118$ GeV and $m_N = 1.020$ GeV. The nucleon mass is not very good. This is an indication that the approximation of a pure $Dq$ structure for these baryons is questionable [12].

For a $J^P = 1/2^+$ $DD\bar{s}$ pentaquark state, a $P$-wave must exist between the two diquarks. So, in good approximation, no instanton interaction can exist between them. We just consider interactions between $\bar{s}$ and the two diquarks. For the pentaquark considered here, we find $\hat{g} = -g'$, $\langle P^{S=0}\rangle = 1/4$, $\langle P^{S=1}\rangle = 3/4$, $\langle P_1^C\rangle = 1/3$ and $\langle P_8^C\rangle = 2/3$. Using the same procedure as for $Dq$ systems, the contribution of the instanton potential for each interaction $D$-$\bar{s}$ is given by

$$\langle DD\bar{s}|V_{\mathrm{inst}}(D\bar{s})|DD\bar{s}\rangle = -3\, g'\, \rho(r_{D\bar{s}})\, \delta_{L,0}. \tag{13}$$

Using these interactions plus the Hamiltonian (4)-(5), we find that the mass of the $1/2^+$ $DD\bar{s}$ pentaquark is 2.241 GeV, which is only 40 MeV below the result obtained in refs. [4,5].

If we ignore our crude trial to obtain a reasonable mass for the $[ud]$ diquark and if we fix arbitrarily $m_D = 0$, then the mass of the $1/2^+$ $DD\bar{s}$ pentaquark is 2.079 GeV. It is then hopeless to reproduce the experimental value of 1.540 GeV by a better fit of the $[ud]$ diquark mass.

We can conclude that, in our model, the instanton-induced interaction does not bring enough attraction in the system to lower the mass of the pentaquark close to the experimental value of 1.540 GeV. When the $[ud]$ diquark mass is fitted on the baryon spectra, the pentaquark mass is found around 2.2 GeV in the diquark picture, with or without the residual instanton attraction [4,5]. Whatever the value taken for the diquark mass, the resulting pentaquark mass is always higher than 2 GeV, as far as a realistic confinement is considered.

Nevertheless, in other works, instanton-based models can predict masses for the pentaquark much closer to the experimental value. If a pentaquark is composed with two different diquarks, one scalar and one tensor, the $P$-wave penalty can be avoided and lower masses can be obtained [15]. It is also possible that the pentaquark is a bound state of a triquark $ud\bar{s}$ and a diquark $ud$ in a relative $P$-wave [16]. In this case, masses in agreement with experiment can also be obtained. In these models, pentaquark masses are computed with mass formulas only. We believe that firmer conclusions must be obtained with dynamical calculations. So, the pentaquark problem certainly deserves further studies.

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